Supplementary Material for Ho, Britz, Delzeit, Leblanc, Roson, Schuenemann and Weitzel "Modelling Consumption and Constructing Long-Term Baselines in Final Demand"

A1. Common consumption models in CGE models

Implementing a demand system that recognizes the full complexity of household consumption behavior noted in section 2 has proved challenging. The tractable demand systems used in many CGE models capture some essential price and income effects but cannot completely depict the non-monotonic dynamics and the full range of cross-price elasticities. Most models employ the constant elasticity of substitution (CES) form, linear expenditure system (LES), or constant difference in elasticity (CDE) demand system as shown in Table A1. (Table A1 is the source of the information given in Figure 5.) More flexible demand systems are complex to implement with few estimates from the empirical literature; the AIDADS, AIDS and translog forms (described below) are used in more limited settings or are in an experimental stage and have yet to become mainstream in large-scale multi-country CGE models.

Section 3.1 summarizes the various functional forms so that we are clear about which parameters are estimated or calibrated, and about which ones are being adjusted over time. In this Appendix we elaborate on some details of these functions so that readers have a convenient comparison between them – their complexity, number of parameters and the ease of implementing them. We provide more references to the literature for those wishing to investigate further.

Table A1. Demand systems currently used in CGE models (results of our survey)

Model; institution/authors	**Key features**	**Consumption function**	**Energy demand treatment**	**Cons. fn features**	**Income elasticity treatment**
ADAGE; RTI	Global or US; myopic; 24 sectors + 10 biofuels	nested CES			
AIM CGE; Japan NIES	Global or national; myopic; 19 sectors + 19 energy	LES	LES or Logit	Food uses FAO projections	η^M adj. over time
C-GEM; Tsinghua	China in Global; myopic	Nested CES		China shares converge to developed countries	
DART; IfW Kiel	Global; myopic;	LES	Mixed Cobb Douglas & CES	η^M from GTAP; use Dellink (2005)	
Envisage; World Bank	Global; myopic; flexible no. of sectors (~30);	CDE and CES; options for LES, AIDADS		use FAO projections	
ENV-Linkages; OECD	Global; myopic; 22 sectors +7 elect.	ELES and CES		params from GTAP	LES params adj. over time
EPPA; MIT JPGC	Global; myopic; 9 sectors + 8 energy + 8 elect.	LES, older version with CES or CDE	Detailed household transportation	CDE for food, converge to rich country shares; detailed household transportation	η^M from Reimer & Hertel (2004)
FARM; US ERS	Global; myopic; 24 sectors + 14 agriculture	LES and CES	Energy services	γ change over time	
G-cubed; McKibbin	Global; foresighted; 12 sectors	CES			

Model; institution/authors	Key features	Consumption function	Energy demand treatment	Cons. fn features	Income elasticity treatment
GDyn; GTAP	Global; myopic; flexible no. of sectors;	CDE and AIDADS	Nested CES in GDyn-E	params from GTAP	CDE parameters not adjusted
GEM-E3; EU JRC	Global; myopic; 31 ectors;	LES; explicit durable stock	Durables linked to energy use	Durables linked to energy use; φ based on GDP/N (-3.5,-1.8)	η^{M}adj. over time
GLOBE; IDS, UK		LES		params from GTAP	
G-RDEM; Britz & Roson 2019	Global, flexible no of sectors, typical 57 or higher (e.g. GTAP-Power)	AIDADS plus CES nests (GTAP-E nests, cereals, meat)	From GTAP-E		AIDADS econometrically estimated
ICES; FEEM		CDE		params from GTAP	
Imaclim-R; CIRED	Global; myopic; 12 sectors	LES	CES for transport services, travel time constraint		
iPETS; NCAR, Boulder	Global; forward-looking	nested CES		params from GTAP	parameters adjusted over time
Mirage; CEPII, Paris		LES; CES: non-subsistence cons.			
MIRAGRODEP; IFPRI					
MSMR; Env Canada	Global; myopic; 20 sectors	nested CES		η from GTAP and Okagawa and Ban (2008)	η^{M} not adjusted, but params adjusted to external proj.

Model; institution/authors	Key features	Consumption function	Energy demand treatment	Cons. fn features	Income elasticity treatment
PACE; ZEW, Mannheim		CES and CD		elasticities from GTAP CDE	
Pheonix; Wing et al	Global; myopic; 27 sectors	CES			
TEA; COPPE, Brazil		CES from EPPA			
USITC; US Intl Trade Comm.		CDE			
Wegener Ctr; U of Graz	Global; myopic; 14 sectors	nested CES	CES		η^M not adjusted
GTM WTO		CDE (GTAP)	CDE		parameters adjusted over time
1-country models					
DYNK; WIFO	Europe; 59 sectors	AIDS	Link to durables		
IGEM; Jorgenson	US; perfect foresight	Translog	Translog		
MONASH, CoPS, Victoria University	Single-country models; 100s sectors				

Source: Documentation and responses to survey of participants of the 2018 OECD/GTAP workshop.

A1.1 Constant elasticity of substitution (CES)

The constant elasticity of substitution (CES) demand system is used in many CGE models (EPPAv6, ADAGE, PACE, etc.). It is relatively simple to implement and only requires an elasticity of substitution parameter (σ) to be chosen, and then share coefficients can be calibrated, at each tier of a nested structure. The utility (within a period t) from consuming commodities c1, …, cn is given by:

$$U_t = \left[\alpha_{1t} c_{1t}^{\frac{\sigma-1}{\sigma}} + \alpha_{2t} c_{2t}^{\frac{\sigma-1}{\sigma}} + .. \; + \; \alpha_{nt} c_{nt}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \qquad (1)$$

The α_{it}'s are parameters that may be calibrated on observed expenditure shares and are indexed by time *t* in models where they are changed exogenously over time. The *t* index is suppressed from here on unless needed for clarity. The budget constraint is expressed as the following for all utility functions discussed in this section:

$$M = \sum_i p_i c_i \tag{2}$$

The demand for good *i* in the CES is a linear function of income, *M*:

$$c_i = \frac{\alpha_i^\sigma M}{p_i^\sigma \sum \alpha_j^\sigma p_j^{1-\sigma}} \tag{3}$$

The income elasticities are simply, $\eta_i^M = 1$, for all periods and levels of incomes. The own-price elasticity given by:

$$\eta_i = -\sigma + \frac{(\sigma - 1)\alpha_i^\sigma p_i^{1-\sigma}}{\sum \alpha_j^\sigma p_j^{1-\sigma}} \tag{4}$$

In the case of unit elasticity ($\sigma = 1$), the CES system becomes the Cobb-Douglas (CD) form where demand is characterized by fixed expenditure shares independent of price or income. This is the easiest to implement, especially when combined with a constant propensity to save, or when a constant share of the consumer budget is used to finance government purchases. In these cases, a single demand system can be formulated to accommodate government, investment and consumption demand.

The CES demand system is almost always implemented as a system of nested CES functions, thus allowing for different substitution relations for different classes of goods. One can have more detailed substitution possibilities between different types of goods within a broader category of goods (e.g. different types of energy or food goods) compared to substitution possibilities between broad categories. Due to the ease of implementation, nested CES utility functions for final household demand are common in models written in MPSGE (Rutherford 1999).

A severe limitation for CES and CD demand systems is the homotheticity of demand, i.e. income elasticities are constrained to be one. This restrictive assumption can be relaxed in LES or CDE demand systems that we describe next. The LES can be seen as an extension to a nested CES demand system, as the CES can be maintained as part of a LES.

A1.2 Linear Expenditure System (LES) or Stone-Geary

The Linear Expenditure System (LES, or ELES if it is "extended" to include savings) allows for non-homothetic demand and is thus one of the most common demand systems applied by both global and national CGE models, including AIM, ENVISAGE, FARM, DART and the IFPRI standard CGE model. The LES utility function is written as:

$$u = \sum_i \alpha_i \ln(c_i - \gamma_i); \quad \sum_i \alpha_i = 1 \tag{5}$$

where γ_i is the commitment level of consumption of good *i*. The term subsistence consumption is often used instead of "commitment" but that is not a very accurate term given that it appears in all consumption items, and it could be negative. The demand for good *i*, the own-price elasticity η_i, income elasticity η_i^M, and number of parameters *np* in a system with *n* goods are derived as:

$$\begin{aligned}
c_i &= \gamma_i + \frac{\alpha_i}{p_i}(M - \textstyle\sum_j p_j\gamma_j) \\
\eta_i &= -\frac{\alpha_i p_i \gamma_i + \alpha_i (M - \sum_j p_j \gamma_j)}{p_i \gamma_i + \alpha_i (M - \sum_j p_j \gamma_j)} = -1 + \frac{(1-\alpha_i) p_i \gamma_i}{p_i c_i} \\
\eta_i^M &= \frac{\alpha_i M}{p_i c_i} \rightarrow 1 \\
np &= 2n - 1
\end{aligned} \tag{6}$$

The Frisch parameter ϕ is the ratio of total to discretionary income and it may be used to calibrate the subsistence parameter and price elasticity:

$$\begin{aligned}
\phi &= -\frac{M}{M - \sum_j p_j \gamma_j} \\
\gamma_i &= c_i + \frac{\alpha_i M}{p_i \phi} \\
\eta_i &= -w_i \eta_i^M \left(1 + \frac{\eta_i^M}{\phi}\right) + \frac{\eta_i^M}{\phi}
\end{aligned} \tag{6b}$$

The commitment demand γ_i is purchased regardless of prices (subject to the minimum income required). The remaining discretionary demand $(c_i - \gamma_i)$ may then be modelled with a CES system or as in (5). The income elasticities η_i^M differ for different goods and the expansion paths are illustrated in Figure A1; (a) shows that if there is no subsistence element, e.g. a pure CES system, the expenditure shares for goods A and B remain unchanged and the expansion path is a straight line through the origin; (b) shows how the subsistence consumption in the LES shifts the origin of the expansion path, and hence changes its slope. In figure A1(b), good A has a higher share of its initial consumption being represented by subsistence consumption and additional income will be spend to a higher share on good B.

(a) (b)

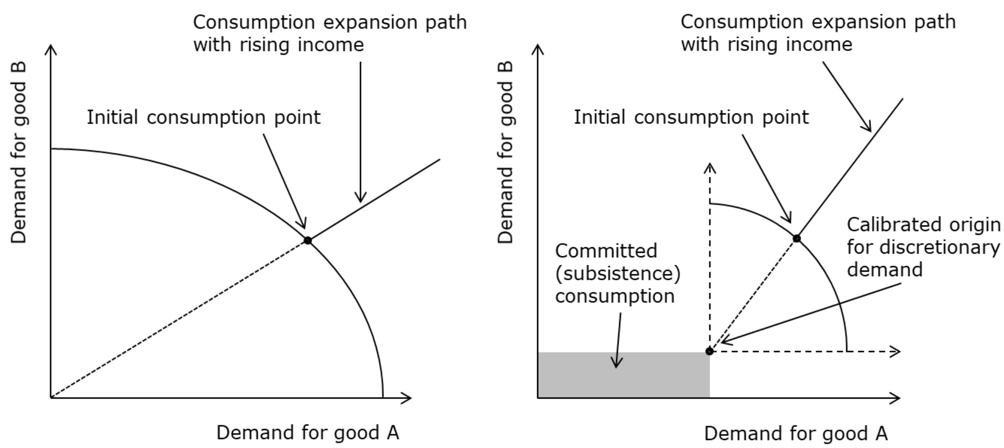

Figure A1. Consumption expansion paths (a) under homothetic CES; (b) non-homothetic LES.

Source: Own elaboration.

If income and price elasticities (η_i^M and η_i) were available, the equations in (6) may be used to calibrate the values of α_i and γ_i. In many cases the price elasticities are not available and some authors would choose a value for the Frisch parameter ϕ , calibrate α_i using some income elasticity, and then calibrate the subsistence parameter (see Annabi et al. 2006 for details). The Frisch parameter is close to -1 for the rich economies with mostly discretionary spending, but larger negative values for poorer countries with a large subsistence share. Frisch proposed a formula to calculate uncompensated own-price elasticities using eq. 6b (Zeitsch et al., 1991) and this approach is used to calculate the GTAP parameters as described by Hertel and van der Mensbrugghe (2016).

Schuenemann and Delzeit (2019) test different ways of calibrating Frisch parameters and subsistence demand and their impact on consumption projections in a dynamic global CGE model. They show that simply using Frisch parameter values from the literature leads to unrealistically high subsistence demand shares. On the other hand, calculating subsistence demand based on a transformation proposed by Dellink (2005)[1] using only information on income elasticities is equivalent to calibrating the Frisch parameter and subsistence demand by solving the above system of equations.

As incomes rise, subsistence consumption become small relative to the discretionary consumption, and the LES converges to a Cobb-Douglas system. This also means that the LES will eventually contradict Engel's law, as the implicit income elasticities approach unity (from above or below).

A1.3 Constant differences of elasticities (CDE)

The Constant Differences of Elasticities (CDE) demand system is used in the standard GTAP model (Corong et al., 2017) and in other models using GTAP data (Envisage, EPPA, ICES, GTM), and represents a more general form of the CES system (van der Mensbrugghe, 2018)[2]. It was first introduced by Hanoch (1975) and can depict non-homothetic preferences as well as non-unitary price elasticities. The expenditure function, *E(p,U)*, underlying the CDE is an implicit indirectly additive function written as follows:

$$\sum_i B_i U^{\beta_i \gamma_i} \left[\frac{p_i}{E(p,U)} \right]^{\beta_i} = 1 \qquad (7)$$

The income elasticities, η_i^M, uncompensated cross-price elasticities, ε_{ij}, and number of parameters are given as:

$$w_i = \frac{Z_i}{\sum_k Z_k}; \quad Z_i = B_i \beta_i U^{\beta_i \gamma_i} (p_i / M)^{\beta_i}$$
$$\alpha_i = 1 - \beta_i$$
$$\eta_i^M = \frac{\sum_k w_k \gamma_k \alpha_k + \gamma_i (1 - \alpha_i)}{\sum_k w_k \gamma_k} + \alpha_i - \sum_k w_k \alpha_k \qquad (8)$$
$$\varepsilon_{ij} = w_j [\alpha_i + \sum_k w_k (1 - \alpha_k) - (1 - \alpha_i)] - \delta_{ij} \alpha_i$$
$$np = 3n$$

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[1] $\gamma_i = \left(1 - \frac{\eta_i}{Max_j\{\eta_j\}}\right) \cdot c_i$

[2] Chen (2017) gives a detailed discussion of the CDE system from which we write eqs. 7 and 8.

w_i are expenditure shares and δ_{ij} is the Kronecker product that equals 1 when $i = j$ and 0 otherwise. γ_i is the expansion parameter and α_i is the substitution parameter.

There are 3n parameters to be estimated or calibrated in the CDE, and they cannot be directly calibrated to demand elasticities given the expressions in (8). The substitution parameters are calibrated to target own-price elasticities, while expansion parameters replicate target income elasticities (Hertel and van der Mensbrugghe, 2016). These substitution and expansion parameters are simultaneously calibrated using maximum entropy methods, but often do not match with the targeted income and own-price elasticities. Chen (2017) shows that a higher sectoral disaggregation of commodities, higher targeted income elasticities and lower targeted price elasticities lead to better matches, as long as target elasticities are valid with respect to Engel/Cournot aggregation and the matrix of Allen-Uzawa elasticities of substitution is negative semi-definite.

These calibration requirements make it difficult to calibrate CDE systems to the large set of empirical income and price elasticities that exist. The CDE system in the standard GTAP model, for example, is indeed calibrated to empirically estimated income elasticities but not to empirical own-price elasticities (Hertel and van der Mensbrugghe, 2016). The own-price elasticities used to calibrate the substitution parameter are calculated using estimated income elasticities following Zeitsch et al. (1991).

As noted in section 3.1, the CDE (like the CES and LES) conserves the calibrated base year income elasticities for all years of the simulation and does not allow for Engel flexibility such as luxury goods becoming necessities (Yu et al., 2004). Section 4.1 discusses the recalibrating of the parameters so that income elasticities may change over time. We next discuss functional forms that allow for Engel flexibility.

A1.4 Flexible demand systems, AIDADS

Flexible demand systems refer to a whole range of more general functional forms. As discussed by Fisher, Fleissig and Serletis (2001), locally flexible forms include the (basic) translog and Almost Ideal Demand System (AIDS) while models with higher rank systems include the EASI, Laurent, QUAIDS, rank-3 translog and General Exponential Form models [3]. An even more general

[3] The rank of a demand system is discussed in Lewbel (1991), and Bouët et al. (2014) explain it as the number of independent price indexes needed to specify the corresponding indirect utility function. Rank 1 systems correspond to homothetic functions with linear Engel curves (e.g. CES); rank 2 have linear Engel curves but do not need to pass through the origin (e.g. LES), rank 3 have non-linear Engel curves. Bouët et al. also summarize the conclusion of Lewbel (1991) as “for average incomes rank 2

approach uses semi-non-parametric forms such as the Fourier and Asymptotically Ideal Model.

One system that allows for endogenous changes in income elasticities is the "An Implicitly Direct Additive Demand System" (AIDADS) that is a rank three system and a generalization of the LES (Preckel et al., 2010, Yu et al., 2000). It is written as an implicit directly additive utility function:

$$\sum_i u_i(c_i, u) = 1$$
$$u_i = \frac{\alpha_i + \beta_i G(u)}{1 + G(u)} \ln\left[\frac{c_i - \gamma_i}{Ae^u}\right] \qquad (9)$$
$$\sum_i \alpha_i = 1 = \sum_i \beta_i$$

γ denotes the commitment consumption as in the LES, and $G(u)$ is a positive monotonic twice differentiable function. As discussed by Yu et al. (2000), when we choose $G(u) = e^u$, the demand function and income elasticities are given by:

$$c_i = \frac{\phi_i (M - \gamma' p)}{p_i} + \gamma_i$$
$$\phi_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u}; \quad \gamma' p = \sum_i p_i \gamma_i \qquad (10)$$
$$\eta_i^M = \frac{\psi_i(c_i, u)}{w_i}; \quad w_i = \frac{p_i c_i}{M}$$
$$np = 3n - 1$$

A, α_i and β_i are parameters to be estimated, w_i is the budget share and ψ_i is the marginal budget share. When $\alpha_i = \beta_i$, the system collapses to the standard LES. There are *3n-1* parameters here compared to *2n-1* for the LES. As the marginal budget shares ψ_i are flexible and individually estimated for low and high incomes, the income elasticities (η_i^M) may vary logistically (Chen, 2017).

The ENVISAGE model includes the option to use the AIDADS demand system and it is used to estimate income elasticities for the GTAP commodities (Hertel and van der Mensbrugghe, 2016; van der Mensbrugghe, 2018). However, calibration for a disaggregated set of commodities is not easy as the system is either underdetermined if it is only calibrated to income elasticities, or overdetermined if calibrated simultaneously to income and price elasticities.

functions are sufficient … but for very low or very high incomes, rank 3 are necessary." Gorman (1981) show that exactly aggregable systems must be rank 3 or less.

Reimer and Hertel (2004) suggest that a maximum of ten commodities might be the practical limit for AIDADS as the direct additivity limits substitution possibilities across a larger number of goods. In Table 6 the other models using AIDADS are G-RDEM and GDyn.

A1.5 Flexible demand systems, AIDS and Translog

AIDADS is a second order flexible income system but not a second order flexible price system. The basic translog or AIDS is a flexible price system which has $\frac{1}{2}n(n+3)-2$ parameters[4]. In the translog system used in Jorgenson et al. (2013 Chap. 3) for a one-country model, the share demand vector for household of type k (w_k) is given as non-linear function of log prices, log income (M) and demographic 1-0 indicator variables (A_k):

$$w_k = \frac{1}{D(p)}(\alpha + B\ln p - B_M \ln M_k + B_A A_k) \qquad (11)$$

$$D(p) = -1 + B_M \ln p; \qquad B_M = B\iota$$

$$np = \tfrac{1}{2}n(n+3) - 2 + \dim(B_A)$$

This is derived from a translog indirect utility function $V(p, M_k)$, where p is the vector of prices. B is the matrix of price coefficients and the vector B_M gives the income effects. B_A is a matrix of coefficients that allow different household types to have different consumption shares even when they face the same prices and have the same incomes. The types of households in Jorgenson et al. include number of adults, number of children, location and race.

The AIDS function has similar cross price terms but is linear in prices:

$$w_i = \alpha_i + \sum_j B_{ij} \ln p_j + \beta_i \ln[y / a(p)] \qquad (12)$$

a(p) is a price index and in the estimation process it is usually set iteratively and not estimated simultaneously.

In flexible systems there is a full set of cross-price elasticities (the B matrix in eq. 11 and 12) and the number of parameters is of order n^2, not counting the demographic terms. The aggregate share demand vector derived by summing over all household types is then a function of the demographic components of the whole population (ξ_t^d), the income distribution (ξ_t^M), aggregate income (M_t), in addition to the usual dependence on prices:

[4] This is discussed by Bouët, Femenia and Laborde (2014).

$$w_t = \frac{1}{D(p_t)}\left[\alpha + B\ln p_t - B_M \frac{\sum M_k \ln M_k}{\sum M_k} + B_A \frac{\sum M_k A_k}{M_k}\right]$$
$$= \frac{1}{D(p_t)}\left[\alpha + B\ln p_t - B_M(\xi_t^M + \ln M_t) + B_A \xi_t^d\right] \quad (13)$$

This translog approach with household type specific parameters thus allows for a full set of price substitutions and allow for a natural way to incorporate projections of demographic changes into the aggregate demand function. The drawback to using such flexible functions is the large number of parameters to be estimated and the need to impose concavity on the B matrix to make it conformable. (An unconstrained B may not be concave, and thus not usable, for prices outside the historical values. That is, parameters that fit the sample period well may generate negative shares with prices that are generated in a long-term projection.) It also limits the number of commodity bundles, and the demand for commodities must be given by a nested structure with something like eq. (13) in the top nest. In Jorgenson et al. (2013) there are 4 bundles in this top tier, while Sommers and Kratena (2017) has 8 items in their AIDS function in DYNK. The complexity of estimating the cross-price elasticities has limited the use of these flexible forms to a few examples of one-region models.

Another limitation of using non-homothetic functions in the top nest is that the functions in the lower tiers must be homothetic for a well-defined price of the sub-aggregate. For example, if the top tier has total energy as a consumption bundle, then a second-tier function allocating energy to electricity, gas, and gasoline must be homothetic. In the system in Jorgenson et al. (2013) the trends observed in the historical data (beyond those captured by price effects) are captured with a state variable since they cannot be a function of income. That is, the changes that are not due to price effects are treated in a way analogous to biases in technical change in production functions.

A1.6 Flexible demand systems, EASI

In response to the widespread use of AIDS in empirical work despite the finding of complex Engel curves, Lewbel and Pendakur (2009) introduced the Exact Affine Stone Index (EASI) form that keeps the linear price form of the AIDS but allow polynomials of income. The budget share of good *i* is given by:

$$w_i = \sum_r b_{ri} y^r + \sum_k (C_{ki} A_k + D_{ki} A_k y) + \sum_{k,j} \Gamma_{kji} A_k \ln p_j + \sum_j B_{ji} \ln p_j y \quad (14)$$
$$y = \log M - \log p' w$$

where A_k is the demographic characteristic indicator as in the translog (11) above, and y is an affine transform of Stone index deflated (log) expenditures.

This function has not been used directly in any CGE model, but is used by Caron et al. (2017) to calibrate the dynamic income effects in their LES model.

A2. Commodity composition of investment demand

We noted in section 5.1 that it is important to model the structure of investment commodity demand well since it is a large share of GDP in many fast growing countries. We also noted that the literature on commodity structure is scarce and here we only review the methods used in current CGE models. This topic is, unfortunately, considered minor in the CGE literature and the documentation of many models reviewed do not even bother to describe how this total investment in allocated.

We first note that different models define Consumption and Investment differently. In the National Accounts there are often three components for Consumption – nondurables, durables and services. Most models label that total as consumption, however, some (e.g. IGEM) classifies consumer durables as Investment. Some countries do not clearly delineate private versus public investment and comparisons across countries or models should be made with care, especially when using parameters from one country for another. Other differences include the speed of adopting the U.N. System of National Accounts treatment of R&D and artistic creations as investment instead of expensing as intermediate purchases.

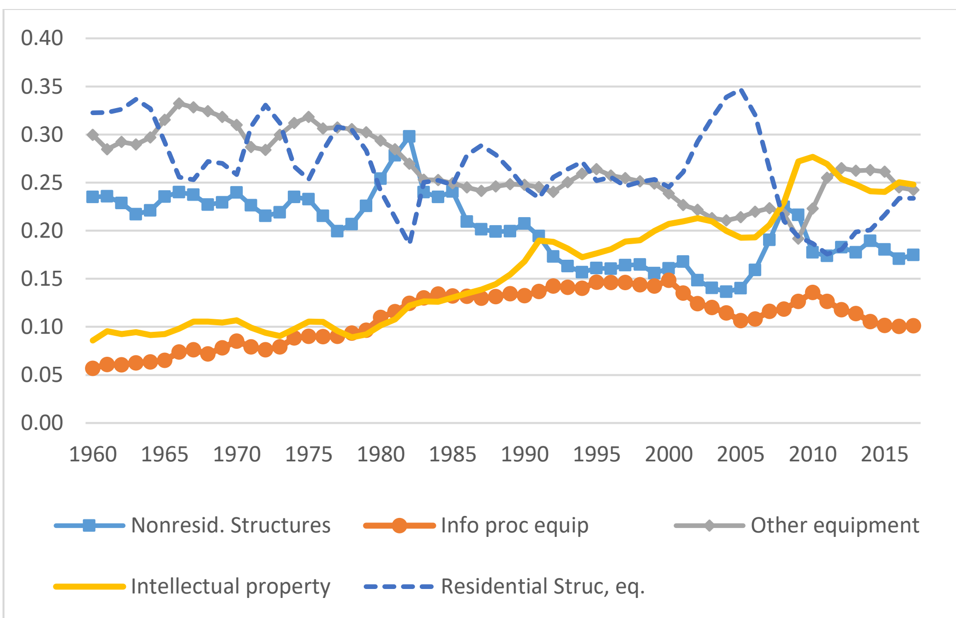

Figure A2. Composition of Investment in the U.S., 1960-2017

Source: Authors construction.

Total investment is the sum of fixed and inventory investment, but we ignore inventory modelling here since its transitory nature makes it unimportant for long range modelling. Figure A2 gives the composition of investment in the U.S. for 1960-2017; intellectual property (including software) investment rose from 5 to 25%, while information processing equipment (e.g. computers) rose steadily to 15% until the dot-com bust and then fell to 10% in 2017. Figure A3 gives the shares for Germany for 1995-2014, where there is a similar rise in intellectual property investment (11% to 17%), and a similar fall in IT equipment after 2000 (7.1% to 3.5%)[5]. Data for other countries compiled for the EUKLEMS show similar big changes in the composition of investment that cannot be wholly explained by price changes; in fact, for much of this period IT prices were falling, and IT investment shares were rising. There is a large literature of factor-biased technical change during the post-1995 period mostly focusing on a switch to skill-intensive technologies, but there is also a switch towards using IT-capital as shown in Figures A2 and A3 and described in Jorgenson et al. (2013, Appendix B).

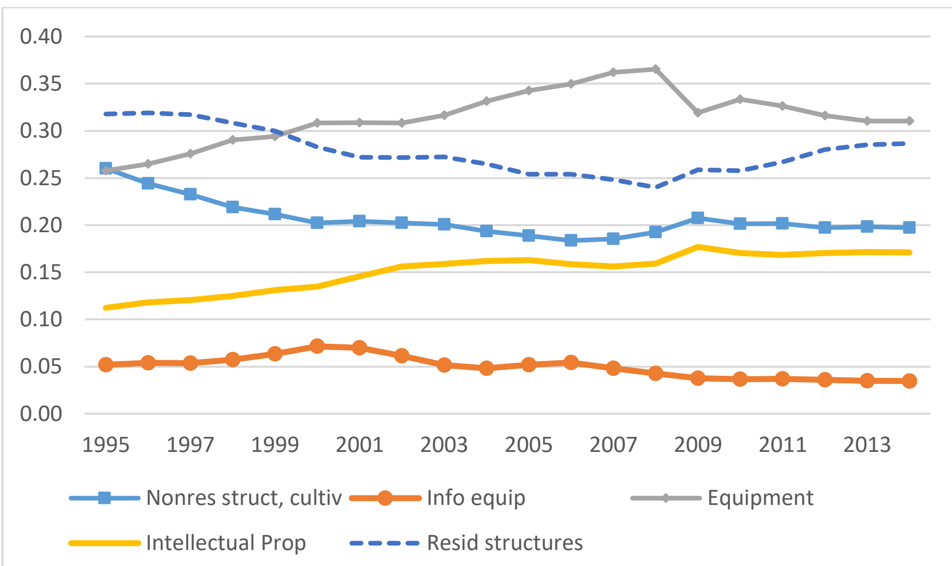

Figure A3. Composition of Investment in Germany, 1995-2014.

Source: Authors construction.

[5] The US data is taken from the National Income and Product Accounts Table 5.3.5 available at https://apps.bea.gov/iTable/index_UD.cfm. The German data is from the EUKLEMS database, at http://www.euklems.net/index_TCB_201701.shtml

In Section 5.1 we noted the discussion in Fouré et al. (this volume) about modeling savings and aggregate investment. The link between consumption and savings, and between savings and investment thus gives a deep link between the modelling of household consumption demand that is the focus of this paper, and the modelling of investment demand.

An example of a simple endogenous savings approach is the utility function in EPPA which depends on total consumption *C* and savings *S*:

$$U_t = U(C_t, S_t) \qquad (18)$$

Intertemporal equilibrium models typically take an Euler equation approach derived from utility functions that are assumed separable over time such as:

$$U = \sum_t^{\infty} \frac{F_t^{\gamma}}{(1+\rho)^t} \qquad (19)$$

where full consumption, F_t, is an aggregate of goods, and possibly leisure, $F_t = F(C_t, L_t)$. The Euler equation then gives savings which drives aggregate investment. Most of the global models, however, use a simple approach with exogenous saving rates.

We do not discuss any of the aspects related to the modelling of this stage of the utility function – how poorly the Euler equation performs, how to interpret the risk aversion parameter, how to implement the discounting, how the separability assumptions are violated, etc[6]. In the remainder of this section we only discuss the allocation of total investment to individual commodities.

We may divide investment allocation models into two broad categories, one that considers only the economy stock of capital and aggregate investment, and one that considers investment by each industry where there is an industry-specific price of the capital stock due to adjustment costs. The models with industry-specific investment includes models with foresight such as G-cubed, and myopic models such as GEM-E3 and Monash. The first approach derives the investment vector for aggregate investment, one can think of this as the Investment column in the Use table. The industry specific investment derives an investment matrix with a column for the capital stock of each industry. However, for our discussion here, the allocation issues for the two approaches are the same and we concentrate on projecting the aggregate Investment vector, $\{I_{it}\}$.

[6] Those interested in empirical work on the Euler equation may start with Canzonieri, Cumby and Diba (2007).

The simplest method is to set I_{it} exogenously, as done in the base version of Lofgren, Harris and Robinson (2002). In other closure settings of that model, this exogenous base investment may be multiplied by a common scale factor so that total investment hits an exogenous savings target. This approach may make sense in a short run model but is not realistic for long term projections as illustrated by the historical trends in Figures A2 and A3.

An easy way to allocate aggregate investment (I^{agg}) is using a Leontief function as in the GTAP model; in the percent change notation of Corong et al. (2017, eq. 42):

$$\widehat{I}_{it} = \widehat{I}_{t}^{agg} \tag{20}$$

This Leontief approach is also used in MONASH (Dixon and Rimmer 2002, Fig. 21.1) and GEM-E3 (Capros et al. 2013 Figure 9).

Other models use a more flexible approach than the exogenous or Leontief formulations. The ENVISAGE and G-cubed models use a CES function of all the component commodities, i.e. with a price elasticity that is common to all commodities. The demand for investment good i is thus a simple function of aggregate investment and its own price, PB_{it}^{I}:

$$I_{it} = \alpha_{it}^{I} \left(\frac{PI}{PB_{it}^{I}} \right)^{\sigma^{I}} I_{t}^{agg} \tag{21}$$

The PB notation denotes the "buyer's price," while PI is the price of the investment bundle (van der Mensburrghe 2008, D-13). This reduces to a simple Cobb-Douglas function when the elasticity parameter, $\sigma^{I} = 1$. The share parameters, α_{it}^{I}, would be calibrated to base year shares, but modelers have to decide on the projection beyond the base year. ENVISAGE (van der Mensburgghe 2009, p 8), for example, fixes the shares at the base year values.

This CES formulation is popular and used in Phoenix (Wing et al. 2011), ADAGE (Ross 2008), EPPA, MIRAGE and TEA. The documentations of these models do not discuss if these share parameters are projected on a path different from base year values. Some models use a nested structure for determining the commodity allocation instead of the flat and symmetrical structure of the Leontief and CES examples above. G-cubed (McKibbin and Wilcoxen, 1999) uses CES functions with a top tier of capital, labor, energy and non-energy bundles.

IGEM has a 5-layer nest of translog functions to allocate aggregate investment to 36 commodities.[7]

Of the models reviewed here, most models seem to fix the share parameters for the projection period. This is understandable given that there is no parallel literature to that estimating income elasticities for consumption – there is little discussion and no consensus about the form of an investment allocation function. One exception is C-GEM, which adjusts the share parameters for China to converge to investment patterns currently observed in other developed countries (Li et al., 2019), using an approach that mirrors the adjustment of share parameters on the consumption side. In the latest G-RDEM version (Britz and Roson 2019), the share parameters of the Cobb-Douglas demand system are shifted depending on per capita income based on the regression using the GTAP data base.

To give an idea of a possible approach that uses historical trends to project investment, we summarize the state-space function used in the single country model IGEM. At the top node, total fixed investment is an aggregate of long-lived and short-lived assets. The short-lived bundle is made up of Equipment-IT and Transportation-Trade-Services; the Equipment-IT bundle is allocated to Machinery, Information Technology and Transportation equipment, $I^{EIT} = I^{EIT}(I^{MACH}, I^{IT}, I^{TRNSP})$; and so on. At each node *m*, a translog price dual function is specified in Kalman filter form:

$$\ln PII_t^m = \alpha^{Im} \ln P_t^{Im} + \tfrac{1}{2} \ln P_t^{Im}{}' B^{Im} \ln P_t^{Im} + \ln P_t^{Im}{}' f_t^{Im} \qquad (22)$$

For example, for *m*=*Equipment-IT*, $PII^{m=EIT}$ denotes the price of the Equipment-IT bundle, $P_t^{I,EIT} = (PII_t^{MACH}, PII_t^{IT}, PII_t^{TRNSP})'$ is the vector of input prices, and $f_t^{I,EIT}$ is latent vector representing the change in technology (or investor preferences). These price functions are estimated over historical data and in the econometric model the unobserved factor is assumed to follow a first-order VAR:

$$f_t^{Im} = F^{Im} f_{t-1}^{Im} + v_t^{Im}$$

The share demand equation derived from the price function (22) is then a linear function of (log) prices and the latent variable:

$$SI^{m=EIT} = \begin{bmatrix} PII_t^{Machinery} I_{Machinery,t}^f / PII^{EIT} I^{EIT} \\ \ldots \\ PII_t^{Services} I_{Services,t}^f / PII^{EIT} I^{EIT} \end{bmatrix} = \alpha^{Im} + B^{Im} \ln P^{Im} + f_t^{Im} t \qquad (23)$$

[7] Jorgenson el al. (2013 p 403) describes an earlier version of IGEM based on the SIC. The current version is based on the NAICS and is described in Jorgenson et al. (2017, section A3).

In the projection period beyond the base year the forecasted series for the latent variable is used. The estimated series of the latent variables for the components of the Information Technology (IT) node are represented in Figure A4, together with the projected values out to 2030. The components are IT-equipment, Publishing & Telecom, and Software & IT-services, and the shares must add to 1. We plot the actual shares in the sample period and the fitted values for two of the 3 components. The fitted values consist of the price and latent terms and together fit the data quite well. In the projection period we plot the matched trend of the latent component of the share function. In the model, the shares are given by (23) with the endogenous price component in addition to this exogenous trend.

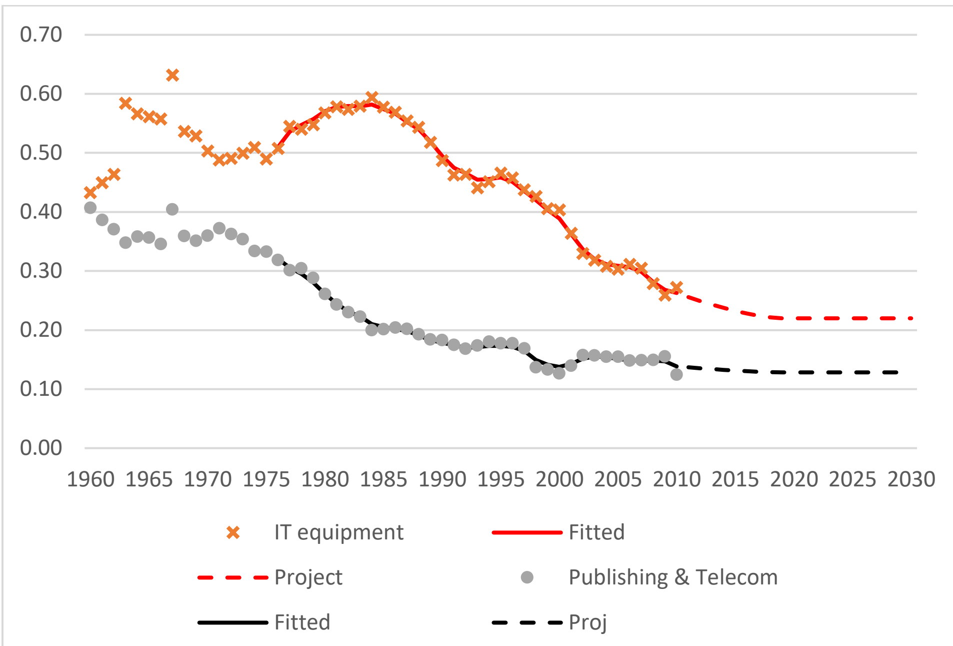

Figure A4. Actual and fitted shares of US investment demand; projected latent term. Node: IT = f(IT equipment, Publishing & Telecom, Software & IT-Services)

Source: Authors construction.

In this example we see a trend for falling share allocated to IT-equipment and a rising share to Software & IT-services. This is due to both changes in relative prices and changes in technology. The econometric model projects a continuing fall in the IT-equipment share due to changes in technology and that is included in the base case projection in IGEM.

The above is just one way of including changes in technology that modelers believe will likely happen. As with consumption modelling, one may prefer a simpler ad-hoc adjustment; in the case of eq. (23) we can change the α^{Im} parameter based on some expert judgement instead of having the econometrically estimated f_t^{Im} term. There are, unfortunately, only limited historical data on investment by commodity for many countries. They may not be in the National Accounts, and modelers have to resort to input-output tables from different years which may not be compiled in a consistent fashion. The data source for Europe used in Figure A3 above for Germany may be the most convenient source of information on investment by commodity. Another source is the World Input-Output Database[8].

A3. Government demand

Aggregate government final demand is specified in various ways in CGE models, but often is not modelled as elaborately as private consumption. Some models do not identify an explicit government sector and combines it with personal consumption expenditures (e.g. PACE, AIM). A simple approach is to allocate a constant share of GDP to government purchases (USITC, GLOBE, ICES). Some models use a Cobb-Douglas function that allocates fixed expenditure shares to private consumption and government consumption (e.g. Wegener Center, Bednar-Friedl et al., 2012). One may emphasize budget constraints (e.g. IGEM) and specify government purchases (G) as a residual in the budget equation with endogenous revenues and exogenous deficits, transfers and interest payments:

$$Deficit = TaxRevenue - G - transfers - interest \qquad (24)$$

Before we describe the data and modelling approaches for government purchases let us first note the differences in public institutions and accounting conventions in different countries. In Section 2 we noted how some countries have a large private Education industry and a corresponding large demand for Education in the Consumption column of the input-output accounts. Other countries may have Government as the dominant source of final demand for Education. Some IO accounts have two distinct industries for private and public education (e.g. the US), while others have one unified column in the Use matrix.

A similar situation holds for Health Services; some countries have a large private health industry while others are dominated by state hospitals. Thus, even if two countries devote a similar share of GDP to the Health sector, the share of Consumption allocated to Health may be very different. In the U.S. accounts we note that even if the hospital bills are paid by the government (through the Medicare program) the expenditure is recorded in the Consumption column, and

[8] This WIOD data is prepared by the University of Groningen and partners and is available at www.wiod.org, see Timmer et al. (2015).

the payments are recorded as transfers from government to the household. Countries with direct government provision of hospitals would record Health expenditures in the Government column. The level and composition of Government final demand is thus different among countries even if the underlying supply share of GDP is similar.

Other accounting differences come from different speeds of adopting the latest U.N. System of National Accounts (SNA). Many countries now include the depreciation of public capital in GDP and government demand, but others have not; some have included public R&D as (public) investment while other keep the old treatment of R&D expenses as intermediate purchases. One should keep all these differences in mind when reading this section.

Figure A5 plots the share of government consumption in GDP over the past decades for G20 members. The top chart shows countries with an annual change exceeding 3 percentage points or a range of decadal average change exceeding 5 percentage points, i.e. countries where the government contribution fluctuates or have pronounced changes. The bottom chart shows countries and country groups without such large changes. Most of the countries in the bottom graph have higher per capita incomes, and we add the low- and middle-income aggregates for comparison.

We draw two conclusions from this figure. First, there is considerable variation between countries for aggregate government consumption, ranging from less than 10% to more than 20% of GDP. Differences in government consumption levels are either driven by structural differences that are unlikely to change substantially over time (Shelton 2007), or driven by different accounting principles. Secondly, there appears to be no relationship between income and the share of government consumption. While India and Indonesia are the poorest G20 members and also characterized with the lowest government consumption share in GDP, this does not hold on a broader level. Figure A5 also plots data for the low and middle-income country aggregates. For recent years, both groups have shares similar to those of the United States.

Second, for most countries, there is surprisingly little variation over time and many countries more or less maintained the expenditure shares and clear trends can be observed for most countries (bottom part of the figure). Countries with larger fluctuations between years (top part of the figure) are mainly countries exposed to international resource prices (Saudi Arabia, Russia). Countries that had bigger changes between decadal average values are those with big changes in the political environment (e.g. Brazil between 1985 and 1990, Argentina in the early 1990s). An assumption of a constant government purchases to GDP ratio may be reasonable for long-run modelling if an endogenous response to policy changes is not required.

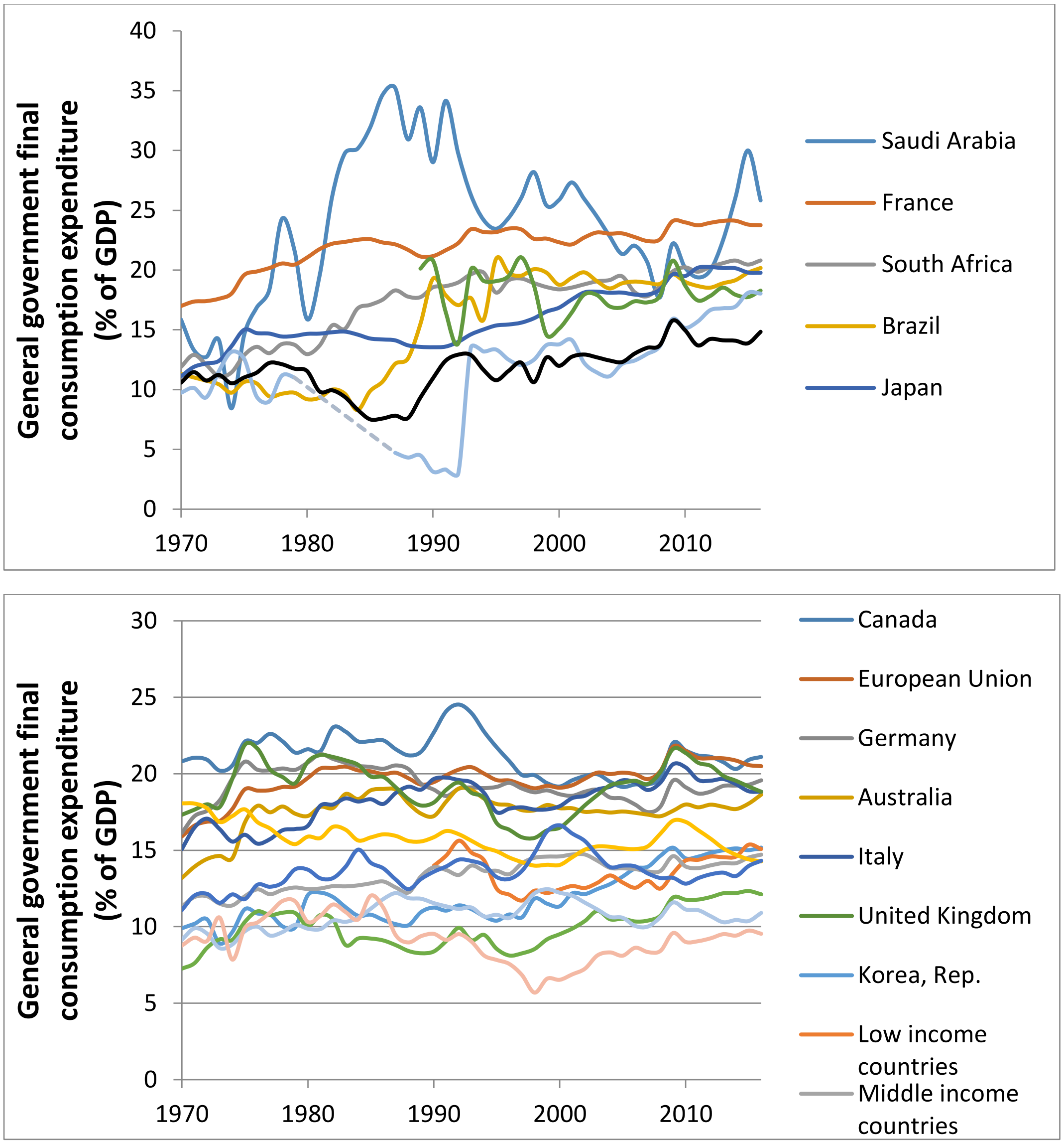

Figure A5. Share of government final consumption as share of GDP

Source: World Bank national accounts data, https://data.worldbank.org/indicator/NE.CON.GOVT.ZS. *The top chart shows countries with an annual change exceeding 3 percentage points or a range of decadal averages exceeding 5 percentage points. The bottom chart shows countries and country groups without such changes.*

A simple approach is to modeling government purchases is thus to allocate a constant share of GDP, which is done by several models. For example, GDyn uses a Cobb-Douglas function that allocates fixed expenditure shares to private consumption and government consumption. Some models do not identify an explicit government sector and combine it with personal consumption expenditures.

The commodity composition of aggregate government purchases is given in different ways in different Input-Output conventions. Some IO tables have a government column in the final demand sector symmetrically with Consumption, other IO tables have a Government Industry that is symmetrical with other industries and a simple Government final demand column that purchase from that Government Industry. The latest US IO system is a mixture of the two with both Government Industries and a final demand column. A model which treats the Government industry symmetrically with Services would be explaining the commodity composition with its chosen production function.

A model that has an explicit government final demand column would have to specify the allocation; all CGE models we surveyed do this is a manner much simpler than the consumption function. Leontief demand, Cobb-Douglas or CES demand are common choices and no model adjusts demand system parameters that we know of. GDyn, ICES, and IGEM models use a Cobb-Douglas function, while the Leontief system is used in GEM-E3, Globe, AIM, USITC, and CES in Envisage, EPPA and TEA.

In the GTAP dataset used by most global models, government purchases are predominantly from the "Public Administration, Defense, Education, Health" sector (Aguiar et al., 2016), which accounts for 94% of all government purchases in the GTAP 9 data for 2011. The commodity composition is thus driven by the production function of that sector. That is, the main mechanism for allocating public purchases in these versions of GTAP is the production function for this large sector; the specification of the final demand function for government is less important since it is dominated by just one sector.

The latest available release of GTAP 10 (Aguiar et al., 2019) disaggregates this sector into three (Public administration; Education; Health and social work). This change from one to three sectors makes the allocation of total government demand more important, in particular if one wishes to incorporate the effect of aging on the demand for health and education.

There is little description of the projection of government commodity allocation in the model documentations; EPPA contains a mention of incorporating an expected rise in the shares for health and education. G-RDEM (Britz and Roson 2019) shifts share parameters of government demand depending on per capita income based on a regression using the GTAP data base, just as in their investment function.

A4. Food demand

Section 2.1.1 discuss the modeling of food demand in some detail. We noted how the shares of different food items vary across countries and time due to income effects and differences in preferences. We cited estimates of income elasticities (Yu et al. 2004 and Cirera and Masset 2010) and noted that as households become richer, elasticities fall and demand for many food items reach a saturation point. In Table A2 we reproduce some estimates of income elasticities for selected countries between 1996 and 2005 (The 1996 estimates are from Seale and Regmi (2006) and the 2005 estimates from Muhammad et al (2011)) . These studies do not have the same coverage and thus the comparisons should be made with this in mind.

Table A2: **Income elasticities of different food types for selected countries and different years**

Food type	Malawi[a]	Vietnam		Mexico		United States	
	2005	1996	2005	1996	2005	1996	2005
Cereals	0.65	0.59	0.54	0.40	0.18	0.05	-0.09
Meats	0.81	0.79	0.77	0.63	0.64	0.10	0.34
Fish	0.72	0.88	0.66	0.67	0.51	0.10	0.26
Dairy	0.84	0.83	0.80	0.65	0.66	0.10	0.35
Oils & Fats	0.66	0.55	0.55	0.32	0.24	0.03	0.00
Fruits & Vegetables	0.70	0.64	0.62	0.49	0.44	0.07	0.21
Other Food	0.98	0.79	1.34	0.63	0.85	0.10	0.44
Beverage & Tobacco	2.85	1.43	1.16	0.80	0.81	0.12	0.42

Notes: [a]No data available for Malawi for 1996.

Source: Seale and Regmi (2006); Muhammad et al. (2011).

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