The appendix C in Lanz and Rutherford (2016, p.47) explains how to calibrate the value of β in the linear expenditure system. It says that β is calibrated by solving the following optimization problem

$$\min_{\beta} \sum_{i} \theta_i \left[\varepsilon_i - \beta\eta_i(\eta_i\theta_i - 1)\right]^2 \tag{1}$$

and thus the following equation determines β

$$\beta = \frac{\sum_i \theta_i\varepsilon_i}{\sum_i \eta_i\theta_i(\eta_i\theta_i - 1)} \tag{2}$$

Actually, in `gtap9data.gms` included in GTAPinGAMS, Eq. (2) is used for determining β.

```
betales(r) = sum(i, thetac(i,r)*varepsilon(i,r))
        / sum(i,eta(i,r)*thetac(i,r)*(eta(i,r)*thetac(i,r)-1));
```

However, the true 1st order condition for the problem (1) is not Eq. (2) but the following equation.

$$\beta = \frac{\sum_i \theta_i\varepsilon_i\eta_i(\eta_i\theta_i - 1)}{\sum_i \theta_i\eta_i^2(\eta_i\theta_i - 1)^2}$$

and Eq. (2) is the 1st order condition for the following problem.

$$\min_{\beta} \sum_{i} \frac{\theta_i}{\eta_i(\eta_i\theta_i - 1)} \left[\varepsilon_i - \beta\eta_i(\eta_i\theta_i - 1)\right]^2$$

I think that there is a slight discrepancy between the problem (1) and its 1st order condition (2).

References

Lanz, Bruno, and Thomas F Rutherford. 2016. "GTAPinGAMS: Multiregional and Small Open Economy Models." *Journal of Global Economic Analysis* 1 (2): 1–77. 10.21642/JGEA.010201AF.